Collective Dynamics and Networks

April 15-16, 2023

Schedule

All times are China Standard Time (GMT+8).

Venue: **Innovation Building 1046**, Duke Kunshan University.

Day 1 (Saturday, April 15)

8:30 - 9:00	**Registration & Welcome**
9:00 - 9:30	**Zhigang Zheng**, Huaqiao University: *Promoting synchrony of power grids by restructuring network topologies*
9:30 - 10:00	**Jiwei Zhang**, Wuhan University: *Dimensional reduction of emergent spatiotemporal cortical dynamics via a maximum entropy moment closure*
10:00 - 10:30	**Jingfang Fan**, Beijing Normal University: *Teleconnections among tipping elements in the Earth system*
10:30 - 11:00	**Break**
11:00 - 11:30	**Xiyun Zhang**, Jinan University: *Epidemic spreading under pathogen evolution*
11:30 - 12:00	**Xingang Wang**, Shaanxi Normal University: *Inferring synchronizability of networked heterogeneous oscillators with machine learning*
12:00 - 13:00	**Lunch**
13:00 - 13:30	**Xiaozhu Zhang**, Tongji University: *Collective dynamic responses and vulnerabilities in complex networks and power grids*
13:30 - 14:00	**Zonghua Liu**, East China Normal University: *How IQ depends on the running mode of brain network?*
14:00 - 14:30	**Break**

14:30 - 15:00	**Huanfei Ma**, Soochow University: *Dynamical Causality: theory and algorithm*
15:00 - 15:30	**Hepeng Zhang**, Shanghai Jiao Tong University: *Collective phenomena in swimming microorganisms*
15:30 - 16:00	**Break**
16:00 - 16:30*	**Marc Timme**, TU Dresden: *Fluctuation-Induced Tipping Points in Complex Systems*
16:30 - 17:00*	**Stefano Boccaletti**, CNR Institute of Complex Systems, Florence: *The transition to synchronization of networked systems*
17:00 - 17:30*	**Jürgen Kurths**, Potsdam Institute for Climate Impact Research & Humboldt University: *Stability in Power Grids and Influences of Climate Extremes*
18:00 - 20:00	**Workshop Dinner**

** means this talk will be delivered online*

Day 2 (Sunday, April 16)

8:30 - 9:00	**Yong Zou**, East China Normal University: *Phase coherence and event coincidence analysis for extracting coupling directions*
9:00 - 9:30	**Muhua Zheng**, Jiangsu University: *The effects of long-range connections on navigation in suprachiasmatic nucleus networks*
9:30 - 10:00	**Zhihong You**, Xiamen University: *Activity-powered liquid-liquid interface*
10:00 - 10:30	**Break**
10:30 - 11:00	**Jinshan Wu**, Beijing Normal University: *Teaching and Learning as Dynamic Process over Concept Networks*

11:00 - 11:30	**Wei Zou**, South China Normal University:*Solvable Dynamics of Coupled High-Dimensional Generalized Limit-Cycle Oscillators*
11:30 - 12:00	**Xiaqing Shi**, Soochow University: *Susceptibility of Orientationally Ordered Active Matter*
12:00 - 13:00	**Lunch**
13:00 - 13:30	**Yueheng Lan**, Beijing University of Posts and Telecommunications: *A variational approach to recurrent patterns in complex systems*
13:30 - 14:00	**Konstantinos Efstathiou**, Duke Kunshan University: *Collective dynamics of second order oscillators*

Titles and Abstracts

Day 1 — Session 1

Promoting synchrony of power grids by restructuring network topologies

Zhigang Zheng
Institute of Systems Science, Huaqiao University, Xiamen 361021, China
College of Information Science and Engineering, Huaqiao University, Xiamen 361021, China
zgzheng@hqu.edu.cn

The optimization of synchronization on distributed power grids is an important topic in recent years. We extensively studied the optimization by restructuring grid topology in terms of connection rewirings. Due to the node-link dual property of the power networks, i.e., the intrinsic generator-load dynamics of nodes and the multiple-attribute connections, we propose the frequency-correlation-optimization scheme to get grid topology with the largest anti-correlation by targeting the frequency correlation function among nodes. The topology optimization on both sparse and dense networks are successfully realized. The optimized topology is shown to possess the vast majority of generator-consumer connections, indicating that a decentralization of the distribution of generator nodes on power grids favors synchronizability. The benefit of these frequency-correlationoptimized power grids to synchrony are verified. By considering the enhancement of the phase coherence among nodes, we further put forward the phase-coherence-optimization scheme, which is applicable to optimize network topology that possesses both the favorable topology and efficient synchronizability. We show that the frequency-correlation optimization and the phase-coherence optimization of power grids are usually reciprocal, while the former is more efficient and simpler in avoiding tedious simulations of high-dimensional nonlinear dynamics. Our explorations may shed light on the predesign and construction of modern distributed power grids, which are composed of decentralized miscellaneous power sources.

Dimensional reduction of emergent spatiotemporal cortical dynamics via a maximum entropy moment closure

Jiwei Zhang
Wuhan University
jiweizhang@whu.edu.cn

While neural population models such as neural mass, population density and kinetic theoretical models have been used to capture a wide range of the experimentally observed dynamics, a full account of how the multi-scale dynamics emerges from the detailed biophysical properties of individual neurons and the network architecture remains elusive. Here we apply a recently developed coarse-graining framework for reduced-dimensional descriptions of neuronal networks to model visual cortical dynamics. We show that, without introducing any new parameters, how a sequence of models culminating in an augmented system of spatially-coupled ODEs can effectively model a wide range of the observed cortical dynamics, ranging from visual stimulus orientation dynamics to traveling waves induced by visual illusory stimuli. In addition to an efficient simulation method, this framework also offers an analytic approach to studying large-scale network dynamics. As such, the dimensional reduction naturally leads to mesoscopic variables that capture the interplay between neuronal population stochasticity and network architecture that we believe to underlie many emergent cortical phenomena.

Teleconnections among tipping elements in the Earth system

Jingfang Fan
Beijing Normal University
jingfang@bnu.edu.cn

Tipping elements are components of the Earth system that may shift abruptly and irreversibly from one state to another at specific thresholds. It is not well understood to what degree tipping of one system can influence other regions or tipping elements. Here, we propose a climate network approach to analyse the global impacts of a prominent tipping element, the Amazon Rainforest Area (ARA). We find that the ARA exhibits strong correlations with regions such as the Tibetan Plateau (TP) and West Antarctic ice sheet. Models show that the identified teleconnection propagation path between the ARA and the TP is robust under climate change. In addition, we detect that TP snow cover extent has been losing stability since 2008. We further uncover that various climate extremes between the ARA and the TP are synchronized under climate change. Our framework highlights that tipping elements can be linked and also the potential predictability of cascading tipping dynamics.

Day 1 — Session 2

Epidemic spreading under pathogen evolution

Xiyun Zhang
Jinan University
zxy_822@126.com

The dynamics of epidemic spreading is often reduced to the single control parameter Ro (reproduction-rate), whose value, above or below unity, determines the state of the contagion. If, however, the pathogen evolves as it spreads, Ro may change over time, potentially leading to a mutation-driven spread, in which an initially sub-pandemic pathogen undergoes a breakthrough mutation. To predict the boundaries of this pandemic phase, we introduce here a modeling framework to couple the inter-host network spreading patterns with the intra-host evolutionary dynamics. We find that even in the extreme case when these two process are driven by mutually independent selection forces, mutations can still fundamentally alter the pandemic phase-diagram. The pandemic transitions, we show, are now shaped, not just by Ro, but also by the balance between the epidemic and the evolutionary timescales. If mutations are too slow, the pathogen prevalence decays prior to the appearance of a critical mutation. On the other hand, if mutations are too rapid, the pathogen evolution becomes volatile and, once again, it fails to spread. Between these two extremes, however, we identify a broad range of conditions in which an initially sub-pandemic pathogen can breakthrough to gain widespread prevalence.

Inferring synchronizability of networked heterogeneous oscillators with machine learning

Xingang Wang
Shaanxi Normal University
wangxg@snnu.edu.cn

In the study of network synchronization, an outstanding question of both theoretical and practical significance is how to allocate a given set of heterogeneous oscillators on a complex network in order to improve the synchronization performance. Whereas methods have been proposed to address this question in the literature, the methods are all based on accurate models describing the system dynamics, which, however, are normally unavailable in realistic situations. Here, we show that this question can be addressed by the model-free technique of a feed-forward neural network (FNN) in machine learning. Specifically, we measure the synchronization performance of a number of allocation schemes and use the measured data to train a machine. It is found that the trained machine is able to not only infer the synchronization performance of any new allocation scheme, but also find from a huge amount of candidates the optimal allocation scheme for synchronization.

Day 1 — Session 3

Collective dynamic responses and vulnerabilities in complex networks and power grids

Xiaozhu Zhang
Tongji University
xiaozhu_zhang@tongji.edu.cn

How units in networked systems dynamically respond to external perturbations and signals essentially underlie the reliable functioning of such systems, examples of which range from neural systems reacting to environmental stimuli to power grids being persistently disturbed by fluctuating power inputs from renewable energy sources. Yet how patterns, characteristics as well as vulnerabilities rise from the collective dynamic responses across networks is far from understood to date. Here we show how fluctuating inputs translate to nontrivially distributed dynamic patterns, heterogeneous stochastic characteristics over network units, and localized vulnerabilities in complex oscillatory networks, e.g. in power grids. We analytically identify three regimes of nonequilibrium stationary response patterns, illustrate how signal's power spectrum dictates the heterogeneity and stationarity of network responses, and also quantify fluctuation-induced vulnerabilities at individual nodes.

How IQ depends on the running mode of brain network?

Zonghua Liu
East China Normal University
zhliu@phy.ecnu.edu.cn

Increasing evidence has shown that intelligence quotient (IQ) depends not only on the aspect of phenomenology such as the size of brain and sexuality but also on the topology of the brain network. We here try to get a deeper understanding by asking how the running mode of the brain network influences IQ. We introduce a parameter to represent the trade-off between wiring cost and processing efficiency and figure out the optimal value of alpha by the approach of network reconstruction. A negative correlation between optimal index alpha and IQ is revealed. To further find out the mechanism for the functional difference between males and females, we move to the local level of brain regions and study the relationship between regional optimal alpha and IQ. We find that the Pearson coefficients of males are significantly different from that of females, including both global and regional levels. These findings show that the functional differences between individuals, including the differences between males and females, are closely related to the different running modes of their brain networks.

Day 1 — Session 4

Dynamical Causality: theory and algorithm

Huanfei Ma
Soochow University
hfma@suda.edu.cn

Data-driven causal quantification and analysis have important applications in fields such as complex systems and nonlinear dynamical systems. In recent years, dynamic causal analysis methods based on embedding theory have gained widespread attention and rapid development. In this report, we will introduce the basics of the dynamic causal framework based on embedding theory, as well as some of our research progress in this field, including dynamic causal methods for short sequence data, causal delay spectrum analysis, and underlying mathematical theory analysis of dynamic causal mechanisms.

Collective phenomena in swimming microorganisms

Hepeng Zhang
School of Physics and Astronomy and Institute of Natural Sciences,
Shanghai Jiao Tong University
hepeng_zhang@sjtu.edu.cn

Swimming microorganisms can develop collective phenomena with extended spatiotemporal coherence through interactions between individual cells. In this talk, I will discuss two recent studies on this subject. First, we carried out experiments with marine algae (Effrenium voratum), which swim in circles at the air-liquid interface, and discovered that effective hydrodynamic repulsion between cells in the far field suppresses density fluctuations and generates disordered hyperuniform states under a wide range of density conditions. The second study focused on bacteria (Paenibacillus vortex) colonies growing on agar plates. In this system, while active turbulence without manifest chirality takes place in the bulk, cells self-organize into a wide, clockwise (viewed from the air side) flow all along the typically tortuous centimeter-scale external boundary. We traced the origin of these robust edge flows back to a weak chiral symmetry breaking mechanism at the individual level. Experimental results in both studies were quantitatively reproduced in simple particle models with hydrodynamic and excluded-volume interactions.

Day 1 — Session 5

Fluctuation-Induced Tipping Points in Complex Systems

Marc Timme
Strategic Professor & Chair for Network Dynamics,
Institute of Theoretical Physics & Center for Advancing Electronics Dresden (cfaed), TU Dresden, Germany
http://networkdynamics.info
marc.timme@tu-dresden.de

We simultaneously rely on the proper function of a breadth of fluctuation-driven nonlinear systems every day, from gene regulation and metabolism in the cell to the distribution of electric power and communication of information. Understanding and predicting the function of interacting nonlinear dynamical systems externally driven out of equilibrium pose major challenges for complex systems, in particular if they exhibit many parameters. Here we report genuinely nonlinear responses to external fluctuations and propose a novel approach to predict the nonlinear responses as well as tipping points at which normal operational states may be lost. Methodologically, we offer a nonlinear response theory with capabilities beyond standard perturbation theory at arbitrarily high order, with application options across fields.

This is work with Malte Schroeder, Moritz Thuemler and others.

[1] Moritz Thuemler et al., IFAC papers online 55:254 (2022).

The transition to synchronization of networked systems

Stefano Boccaletti
CNR Institute of Complex Systems, Florence
stefano.boccaletti@gmail.com

From brain dynamics and neuronal firing, to power grids or financial markets, synchronization of networked dynamical units is the collective behavior characterizing the normal functioning of most natural and man made systems.

As an order parameter (typically the coupling strength in each network's link) increases, synchronization occurs via a transition between a fully disordered and gaseous-like phase (where the units evolve in a totally incoherent manner) to an ordered or solid-like phase (in which, instead, all units follow the same trajectory in time).

The transition between such two phases can be discontinuous and irreversible, or smooth, continuous, and reversible. The first case is known as Explosive Synchronization, which has been described in various circumstances, and which refers to an abrupt onset of synchronization following an infinitesimally small change in the order parameter, with hysteresis loops that may be observed as in a thermodynamic first-order phase transition. The second case is the most commonly observed one, and corresponds instead to a second-order phase transition, resulting in intermediate states emerging in between the two phases. Namely, the path to synchrony is here characterized by a sequence of events where structured states emerge made of different functional modules (or clusters), each one evolving in unison.

With the only help of eigenvalues and eigenvectors of the graph's Laplacian matrix, I will show that the transition to synchronization of a generic networked dynamical system can be entirely predicted and completely characterized. In particular, I will report that the transition is made of a well defined sequence of events, each of which corresponds to either the nucleation of one (or several) cluster(s) of synchronized nodes or to the merging of multiple synchronized clusters into a single one. The network's nodes involved in each of such clusters can be exactly identified, and the value of the coupling strength at which such events are taking place (and therefore, the complete events' sequence) can be rigorously ascertained. I will moreover clarify that the synchronized clusters are formed by those nodes which are indistinguishable at the eyes of any other network's vertex, and as so they are receiving the same dynamical input from the rest of the network. Therefore, such clusters are more general subsets of nodes than those defined by the graph's symmetry orbits, and at the same time more specific than those described by the network's equitable partitions. Finally, I will present large scale simulations which show how accurate the predictions are in describing the synchronization transition of both synthetic and real-world large size networks, and I even report that the observed sequence of clusters is preserved in heterogeneous networks made of slightly non identical systems.

Stability in Power Grids and Influences of Climate Extremes

Jürgen Kurths
Potsdam Institute for Climate Impact Research and Humboldt University Berlin, Institute of Physics, Germany
juergen.kurths@pik-potsdam.de

Power grids as well as several other real-world systems are characterized by multistability. For power grids, the strongly ongoing transition to distributed renewable energy sources, to green energy, leads to a proliferation of dynamical actors. The desynchronization of a few or even one of those would likely result in a substantial blackout. Thus, the dynamical stability of the synchronous state has become a crucial topic in power grid research.

Here we claim that the traditional linearization-based approach to stability is in several cases too local to adequately assess how stable a state is. Instead, we quantify it in terms of basin stability, a measure related to the volume of the basin of attraction. Basin stability is non-local, nonlinear and easily applicable, even to high-dimensional systems, as power grids.

Remarkably, when taking physical losses in the network into account, the back-reaction of the network induces new exotic solitary states in the individual actors, and the stability characteristics of the synchronous state are dramatically altered. These novel effects will have to be explicitly considered in the design of future power grids, and their existence poses a challenge for control.

Considering the vulnerability of power grids against extreme climate events, in particular wind loads and, consequently, increasing its robustness to withstand these events is of great importance. Here, we combine a detailed model of the climatic drivers of extreme events, and a cascadable model of the transmission network to provide a holistic co-evolution model to consider wind-induced failures of transmission lines in the Texan electrical network. This way most vulnerable connections are identified.

References:

P. Menck, J. Heitzig, N. Marwan, and J. Kurths, Nature Physics 9, 89 (2013)

P. Menck, J. Heitzig, J. Kurths, and H. Schellnhuber, Nature Communication 5, 3969 (2014)

F. Rodrigues, T. Peron. P. Ji, J. Kurths, PHYSICS REPORTS-REVIEW SECTION OF PHYSICS LETTERS 610, 1-98 (2016)

F. Hellmann, P. Schultz, P. Jaros, R. Levchenko, T. Kapitaniak, J. Kurths, and Y. Maistrenko, Nature Communications 11, 592 (2020)

Yang, Z., Yu, J., Kurths, J., & Zhan, M. IEEE Journal on Emerging and Selected Topics in Circuits and Systems, 11(1), 5-16 (2021)

D. Witthaut, F. Hellmann, J. Kurths, S. Kettemann, H. Meyer-Ortmanns, M. Timme, Review Modern Physics 94, 015005 (2022)

A. Büttner, J. Kurths, F. Hellmann, New J. Phys. 24, 053019 (2022)

M. Anvari et al., submitted (2023)

Day 2 — Session 1

Phase coherence and event coincidence analysis for extracting coupling directions

Yong Zou
East China Normal University
yzou@phy.ecnu.edu.cn

The phase coherence analysis was originally introduced to understand interactions between two weakly coupled chaotic oscillators more two decades ago. In the recent years, it has found wide applications in climate sciences, for example, identifying phase coherence between ENSO and the Indian monsoon, and that between the South American monsoon system and Rossby waves. However, to the best of our knowledge, the phase locked plateau length has not been fully considered in the traditional event synchronization analysis, which instead regards the entire plateau as one single event. The explicit inclusion of phase loacked plateaus helps to quantitatively characterize the synchronization levels between different locations. Furthermore, we propose generalized event synchronization analysis for all plateaus and extreme dry events by co-occurence rate and precursor rate of phase locked plateaus. We will show an application to understand the teleconnections between the surrounding oceans and precipitation in northeastern Brazil.

The effects of long-range connections on navigation in suprachiasmatic nucleus networks

Muhua Zheng
Jiangsu University
zhengmuhua163@gmail.com

In the mammalian suprachiasmatic nucleus (SCN), the cellular oscillators communicate within a neuronal network to regulate the circadian rhythms of physiological and behavioral activity. However, how the network topology drives the information communication in the SCN remains poorly understood, especially for the effects of long-range connections. In this talk, we investigate the roles of long-range connections in navigation in spatially embedded SCN networks. First, we show that the SCN networks can be embedded into hidden hyperbolic metric spaces with a simple connectivity law, and the hyperbolic distances can measure the long-range connections. Second, we present that after removing a small fraction of connections with long hyperbolic distances, the residual SCN network can still maintain the robustness of the system and relatively efficient information communication. In contrast, the network becomes very vulnerable after removing the same fraction of connections with long distances in Euclidean space. The navigability between left and right SCN is seriously damaged. The roles of short- and long-range connections are virtually indistinguishable in Euclidean space. Third, we adopt a biophysical model to verify the findings in navigation and show that highly efficient navigation implies the system can sustain the circadian rhythm better. Our results suggest that long-range connections in Euclidean space are undoubtedly meaningful in maintaining the stability and navigability of a system. In contrast, distances in the underlying hyperbolic space can provide almost perfectly navigable maps of SCN networks.

Activity-powered liquid-liquid interface

Zhihong You
Xiamen University
zhyou@xmu.edu.cn

Interfaces and membranes are ubiquitous in cellular systems across various scales. From lipid membranes to the interfaces of biomolecular condensates inside the cell, these borders not only protect and segregate the inner components from the outside world, but also are actively participating in mechanical regulation and biochemical reaction of the cell. Being part of a living system, these interfaces (membranes) are commonly active and away from equilibrium. Yet, it's still not clear how activity can tweak their equilibrium dynamics. Here, I will introduce a model system to tackle this problem. We put together a passive fluid and an active nematics, and study the behavior of this liquid-liquid interface. Whereas thermal fluctuation of such an interface is too weak to be observed, active stress can easily force the interface to fluctuate, overhang, and even break up. In the presence of a wall, the active phase exhibits superfluid-like behavior: it can climb up walls -- a phenomenon we call activity-induced wetting. I will show how to formulate theories to capture these phenomena, highlighting the nontrivial effects of active stress. Our work not only demonstrates that activity can introduce interesting features to an interface, but also sheds light on controlling interfacial properties using activity.

Day 2 — Session 2

Teaching and Learning as Dynamic Process over Concept Networks

Jinshan Wu
Beijing Normal University
jinshanw@bnu.edu.cn

Knowledge can be modeled as a concept network where nodes are concepts and links are logical connections among concepts. In this sense, learning becomes a dynamic process over this concept network. With this model, we aim to answer the core questons of teaching and learning, what to teach and learn and how to teach and learn. For this, we have proposed algorithms on identifying key nodes and key links, finding possibly personalized optimal learning orders, and diagnozing misconceptions. Preliminary results will be reported in this presentation. This also shows how significantly the field of systems science may contribute to education.

Solvable Dynamics of Coupled High-Dimensional Generalized Limit-Cycle Oscillators

Wei Zou
South China Normal University
zouwei@m.scnu.edu.cn

We introduce a new model consisting of globally coupled high-dimensional generalized limit-cycle oscillators, which explicitly incorporates the role of amplitude dynamics of individual units in the collective dynamics. In the limit of weak coupling, our model reduces to theD-dimensionalKuramoto phase model, akin to a similar classic construction of the well-known Kuramoto phase model from weakly coupled twodimensional limit-cycle oscillators. For the practically important case of $D = 3$, the incoherence of the model is rigorously proved to be stable for negative coupling ($K < 0$) but unstable for positive coupling ($K > 0$); the locked states are shown to exist if $K > 0$; in particular, the onset of amplitude death is theoretically predicted. For $D \geqslant 2$, the discrete and continuous spectra for both locked states and amplitude death are governed by two general formulas. Our proposed D-dimensional model is physically more reasonable, because it is no longer constrained by fixed amplitude dynamics, which puts the recent studies of theD-dimensionalKuramoto phase model on a stronger footing by providing a more general framework forD-dimensional limit-cycle oscillators

Susceptibility of Orientationally Ordered Active Matter

Xiaqing Shi
Department of Physics, Soochow University, 215006, Suzhou
xqshi@suda.edu.cn

We investigate the susceptibility of long-range ordered phases of two-dimensional dry aligning active matter to population and substrate spatial quenched disorder. The population disorder is taken in the form of a distribution of intrinsic individual chiralities. Using a combination of particle-level models and hydrodynamic theories derived from them, we show that while in finite systems all ordered phases resist a finite amount of such chirality disorder, the homogeneous ones (polar flocks and active nematics) are unstable to any amount of disorder in the infinite-size limit. For spatial quenched disorder, we find, for random coupling processes, the polar ordered phase remains long-range ordered, but qualitatively different from the pure (disorderless) case. For random scatterers, polar order varies with system size but has strong non-self-averaging, with sample-to-sample fluctuations dominating asymptotically, which prevents us from elucidating the asymptotic status of order. In summary quenched disorder affects active matter system in ways more complex and far reaching than intuitive expectations, and polar order state is extremely fragile to such disorders.

Day 2 — Session 3

A variational approach to recurrent patterns in complex systems

Yueheng Lan
Beijing University of Posts and Telecommunications
lanyh@bupt.edu.cn

Recurrent patterns play essential roles in the study of complex systems and hence an efficient way of recovering them has long been focus of research. To cope with the instability in the dynamics and the orbital complexity of these cyclic structures, we design a variational method for their determination, which pushes a guess loop with proper topology to the target cycle. Recently, a reduced scheme is practiced in this variational approach, achieving great reduction in both storage requirement and computation time for treating high-dimensional systems.

Entrainment of Winfree oscillators under external forcing

Konstantinos Efstathiou
Duke Kunshan University, Zu Chongzhi Center for Mathematics and Computational Science
k.efstathiou@dukekunshan.edu.cn

In this talk we present recent work on the collective dynamics of coupled Winfree oscillators. In particular, we consider globally connected coupled Winfree oscillators under the influence of an external periodic forcing as a model for circadian synchronization. We describe the system's collective dynamics and, in particular, its capability of entrainment to the external forcing. The latter is quantified through the entrainment degree, that is, the proportion of oscillators that synchronize to the forcing. We show how the entrainment degree depends on the inter-oscillator coupling strength, external forcing strength, and the distribution of natural frequencies of the Winfree oscillators, and we compare the results for the different cases. Finally, we compute an approximation of the entrainment degree using a mean-field method, based on the Ott-Antonsen Ansatz. This is joint work with Igor Hoveijn and Yongjiao Zhang.